Online Number Theory Seminar

14 April 2023. – 17:00-17:50

László Szalay: On the difference of k-generalized Lucas numbers and powers of 2

Assume that $k \geq 2$ is a given positive integer. The k-generalized Lucas sequence $\{L_n^{(k)}\}_{n\geq 0}$ has positive integer initial values k, 1, 3, …, $2^{k-1}-1$, and each term afterward is the sum of the k consecutive preceding elements:

$$L_n^{(k)} = L_{n-1}^{(k)} + L_{n-2}^{(k)} + \cdots + L_{n-k}^{(k)}.$$

An integer n is said to be close to a positive integer m if it satisfies $|n-m| < \sqrt{m}$.
In the talk, we solve completely a closeness problem, namely the diophantine inequality

$$\left|L_n^{(k)} - 2^m\right| < 2^{m/2}$$

in the non-negative integers k, n, and m. This problem is equivalent to the resolution of the equation $L_n^{(k)} = 2^m + t$ with the condition $|t| < 2^{m/2}$, $t \in \mathbb{Z}$. We also discovered a new formula for $L_n^{(k)}$ which was very useful in the investigation of one particular case of the problem.