

Online Number Theory Seminar

16 October 2026. – 17:00-17:50 CEST

R. Scott & R. Styer: Recent Advances on the number of solutions to the ternary purely exponential Diophantine equation $a^x + b^y = c^z$.

We summarize recent results on $N(a, b, c)$, the number of solutions in positive integers (x, y, z) to the equation $a^x + b^y = c^z$, for integers a, b, c all greater than one and not necessarily relatively prime. Currently all cases of $N(a, b, c) > 2$ are known and have been shown to be the only such cases. This was done for the case $\gcd(a, b) = 1$ as of 2024; we give a brief summary of the steps leading to this result, and also show how the methods used apply to the more general equation $X + Y = c^z$, where c is fixed and the prime divisors of X and Y are known. The case of $N(a, b, c) > 2$ for $\gcd(a, b) > 1$ was not completely handled until recently, so we give a brief outline of the proof.

It remains to show that the known cases with $N(a, b, c) = 2$ are the only such cases. There are several infinite families of such cases, and 21 anomalous cases with $N(a, b, c) = 2$ that are apparently not in any such infinite family. A current goal is to show that these 21 cases are the only such cases. We give a brief survey of recent work towards this goal, along with some further observations.