Online Number Theory Seminar

29 May 2026. – 17:00-17:50

Sz. Révész: Recent advances in the theory of Beurling systems (a survey).

Beurling initiated the study of generalised number systems in 1936. His main interest was to analyse, in what extent the fundamental theorem of arithmetic – which can be rephrased as the Euler formula for the respective zeta function – allows to draw conclusions about the distribution of generalised primes. He proved that a surprisingly light asymptotic condition on the number of generalised integers $N(x)$ suffices for the PNT (Prime Number Theorem): $N(x)-x = O(x/log^a x)$ suffices if $a > 3/2$ - and does not suffice, if $a = 3/2$ or smaller.
The theory of Beurling number systems then developed to a test field, where various hypothesis, their strength and limitations, could be tested. Diamond, Montgomery and Vorhauer showed in the early 2000's that even if $N(x) - x$ is assumed to be quite small, like square-root of x, it is still possible that the Beurling zeta function has roots as close to the 1-line as $c/\log t$, and nothing better than the classical de la Vallée Poussin error term holds in the PNT.
The lecture will survey the theory with an emphasis of recent developments which considerably enriched our knowledge. We try to distinguish between phenomena which generalise what we seem to know about the classical case of the Riemann zeta function, and phenomena which are new, and cannot be formulated or expected for the classical case.