Online Number Theory Seminar

1 December 2023. – 17:00-17:50

L. Remete:

Comparing equivalences of polynomials: An addendum to the talk of K. Győry on reduction theory of integral polynomials. (Short survey and some joint results with M. Bhargava, J.-H. Evertse, K. Győry and A. Swaminathan)

The purpose of this talk is to demonstrate the relations of the different types of equivalences of integral polynomials occurring in the literature. Classical finiteness results were obtained by Lagrange (1773) and Hermite (1851) in the quadratic and cubic cases. Hermite attempted to generalize his result to the case of arbitrary degree, but he was not successful. Instead, he defined a weaker equivalence for which he could generalize his finiteness theorem to any degree. Hermite's original objective, the finiteness results for the classical equivalences, was finally achieved more than a century later by Birch and Merriman (1972) and independently, for monic polynomials and in effective form by Győry (1973). These results are based on modern deep tools that were unavailable to Hermite. This suggests that the equivalence defined by Hermite is much weaker than the classical $GL_2(\mathbb{Z})$ - and \mathbb{Z} -equivalences. Indeed, to show this difference, we present infinite families of Hermite equivalence classes of integral polynomials that split into at least two $GL_2(\mathbb{Z})$ equivalence classes.

This talk is the second part of a series of lectures. The first part was given by K. Győry on 24th November about the reduction theory of integral polynomials with given discriminant. The third part will be delivered by J.-H. Evertse on 8th December on his new general results on rationally monogenic orders and orders with few rational monogenizations.