Online Number Theory Seminar

17 February 2023. – 17:00-17:50

Robert Tichy: Diophantine Equation of the Pillai Type: Extensions and Applications

After a short survey on well-known results on Pillai's equation $a^x - b^y = c$ we present extensions involving linear recurrences (U_n) . In particular, we show under mild technical conditions that the equation $U_n - b^m = c$ has at most 2 solutions (n, m) for $b > B, n > N_0$ (effectively computable, joint work with Heintze, Vukusic and Ziegler). Furthermore, we consider Pillai's density problem, obtaining asymptotic results for the number of $c \in [1, x]$ such that $c = U_n - V_m$ for some $n, m \in \mathbb{N}$, (joint work with Vukusic, Yang and Ziegler). Other results are devoted to the so-called Pillai-Tijdeman equation. In the final section of the talk a new application of diophantine number theory to purely algebraic problems is discussed. It is shown that the binomial polynomials are absolutely irreducible in the ring Int \mathbb{Z} of integer-valued polynomials over \mathbb{Q} . This result is due to Rissner and Windisch and involves linear algebra tools as well as results on Pillai type equations and elementary prime number theory.