Online Number Theory Seminar

17 October 2025. – 17:00-17:50

A. Pethő: Common values of linear recurrences related to ABC polynomials.

Let $P_u(X) = X \cdot Q(X) \cdot (X-u) + 1$ with $Q(X) \in \mathbb{Z}[X]$ and with the non-negative integer parameter u. Such kind of polynomials seems to be investigated at the first time by Ankeny, Brauer and Chowla in 1956.

One can associate to any integer polynomial a Thue equation. Denote the degree of $Q(X)$ by q then it is

$$Y^{q+2} \cdot P_u(X/Y) = X \cdot (Y^q \cdot Q(X/Y)) \cdot (X-uY) + Y^{q+2} = 1. \tag{1}$$

Halter-Koch, Lettl, Pethő and Tichy, 1999, solved (1) essentially with $Q(X) = \prod_{j=1}^{q}(X-a_j)$ and $a_1, \ldots, a_q$ pairwise distinct integers under the assumption of the Lang-Waldschmidt conjecture.

One can associate to a polynomial not only a Thue equation, but a linear recursive sequence too. With Tengely we studied recently the sequence associated to Shank's simplest cubics. Denote $(A_n(u))$ the linear recursive sequence, which have the initial terms $A_0, \ldots, A_{q+1} \in \mathbb{Z}$, and whose characteristic polynomial is $P_u(X)$. In the talk we show a general effective theorem on the solutions $n, m, u \in \mathbb{Z}, n \neq m$ of the equation

$$|A_n(u)| = |A_m(u)|. \tag{2}$$

My dream was to prove such result for the original ABC polynomials, but my method works only if $Q(0) = \pm 1$.