H. Pasten: The greatest prime factor of $n^2 + 1$

Let $P(k)$ be the greatest prime factor of the integer k. It is an old problem dating back to the 30's to give lower bounds for $P(n^2 + 1)$. This number $P(n^2 + 1)$ can be smaller than n (for instance with $n = 117$ we get $P(n^2 + 1) = 37$) and the problem is to show that it cannot be too small. Work of Chowla from 1934 gave the lower bound $P(n^2 + 1) \gg \log\log n$ and since then, not much progress has been obtained. In this talk I will outline the proof of a lower bound roughly of size $(\log\log n)^2$. In addition I will give improved bounds in subexponential (unconditional) results towards the ABC conjecture. The main ingredients come from two different worlds: transcendence theory, and arithmetic of Shimura curves.