Online Number Theory Seminar

2 June 2023. – 17:00-17:50

Paul Voutier : A kit for linear forms in three logarithms

Lower bounds for linear forms in logarithms are a powerful tool that have found application to many number theory problems. In fact, many problems can be reduced to linear forms in two or three logarithms. Good lower bounds for such linear forms have resulted in the complete solution of several outstanding problems [1, 2].

In the early 2000s, Mignotte produced his "A kit on linear forms in three logarithms" manuscript. This played a crucial role in [2], determining all perfect powers in the Fibonacci sequence. This kit has also circulated in manuscript form since then.

Recently, Mignotte and I have undertaken the task of making this kit manuscript ready for publication. This work is now complete and in this talk I discuss this work. This work also includes several improvements to the initial manuscript.

As a demonstration of our improvements, and to provide a fully worked example that others can follow for application of this kit to their own problems, we rework the lower bounds for the linear form in [2] used to show that there is no solution of $y^p = F_n$ for $n > 12$. We obtain an upper bound on p that is nearly 10 times smaller than the one obtained in [2].

Pari/GP code for the application of the kit, along with examples, is also publicly available at `https://github.com/PV-314/lfl3-kit`. Researchers have already found it helpful (and easy to use!) for addressing several diophantine problems. Joint work with Maurice Mignotte.

References

[1] Y. Bilu, G. Hanrot and P. M. Voutier (with an appendix by M. Mignotte), *Existence of Primitive Divisors of Lucas and Lehmer Numbers*, Crelle's J. **539** (2001), 75–122.

[2] Y. Bugeaud, M. Mignotte and S. Siksek, *Classical and modular approaches to exponential Diophantine equations I. Fibonacci and Lucas perfect powers*, Ann. Math. **163** (2006), 969–1018.