Online Number Theory Seminar

14 October 2022. – 17:00-17:50

Maciej Ulas: Values of certain binary partition function represented by sum of three squares

Let m be a positive integer and $b_m(n)$ be the number of partitions of n with parts being powers of 2, where each part can take m colors. We show that if $m = 2^k - 1$, then there exists the natural density of integers n such that $b_m(n)$ can not be represented as a sum of three squares and it is equal to $1/12$ for $k = 1, 2$ and $1/6$ for $k \geq 3$. In particular, for $m = 1$ the equation $b_1(n) = x^2+y^2+z^2$ has a solution in integers if and only if n is not of the form $2^{2k-1}(8s+2t_s+3)+i$ for $i = 0, 1$ and k, s are positive integers, and where t_n is the nth term in the Prouhet-Thue-Morse sequence.