

Online Number Theory Seminar

18 March 2022. – 17:00-17:50

István Pink: Number of solutions to a special type of unit equations in two unknowns

For any fixed coprime positive integers a, b and c with $\min\{a, b, c\} > 1$, we prove that the equation $a^x + b^y = c^z$ has at most two solutions in positive integers x, y and z , except for one specific case which exactly gives three solutions. Our result is essentially sharp in the sense that there are infinitely many examples allowing the equation to have two solutions in positive integers. From the viewpoint of a well-known generalization of Fermat's equation, it is also regarded as a 3-variable generalization of the celebrated theorem of Bennett [M.A.Bennett, On some exponential equations of S.S.Pillai, *Canad. J. Math.* 53(2001), no.2, 897–922] which asserts that Pillai's type equation $a^x - b^y = c$ has at most two solutions in positive integers x and y for any fixed positive integers a, b and c with $\min\{a, b\} > 1$. In this talk we give a brief summary of corresponding earlier results and present the main improvements leading to this definitive result. This is a joint work with T. Miyazaki.