Online Number Theory Seminar

7 June 2024. – 17:00-17:50

N. Hirata-Kohno: Number of the solutions of S-unit equation in two variables

This is a joint work with Makoto Kawashima, Anthony Poels and Yukiko Washio. We apply the explicit Pade approximation constructed for binomial functions by the second and the third authors, to give a new upper bound for the number of the solutions of the S-unit equation, that refines the bound due to J.-H. Evertse.

Let K be a number field of degree m and let a, b be non-zero elements of K. Consider a finite set S of places of K containing all the Archimedean ones. Denote by s its cardinality and by U_S the set of the S-units in K. In 1984, Evertse proved that the S-unit equation $ax + by = 1\,(x, y \in U_S)$ has at most $3 \times 7^{m+2s}$ solutions.

We refine for any positive integers m, s showing that the equation $ax + by = 1$ has at most $(3.1 + 5(3.4)^m) \times 45^s$ solutions $(x, y) \in U_S^2$.

We use the result proven by Loher and Masser in 2004 to obtain a further improvement: $(3.1 + 68m \log m(1.5)^m)45^s$, which is smaller than the bound above when $m \geq 6$ and $s \geq 1$.