Online Number Theory Seminar

9 June 2023. – 17:00-17:50

Hamid Ben Yakkou : Recent results in the study of monogenity and indices in number fields

Let $K = \mathbb{Q}(\theta)$ be a number field of degree n with θ a root of a monic irreducible polynomial $F(x)$ in $\mathbb{Z}[x]$, and $\mathbb{Z}_K$ its ring of integers. The field K is monogenic if $\mathbb{Z}_K$ admits a power integral basis of the form $(1, \eta, \ldots, \eta^{n-1})$ for some primitive element $\eta \in \mathbb{Z}_K$, that is $\mathbb{Z}_K = \mathbb{Z}[\eta]$. The fundamental method to test whether K is monogenic or not and determine all the power integral bases is to solve an index form equation $I(x_2, \ldots, x_n) = \pm 1$ relative to an integral basis of K. For index form equations, there are general effective finiteness results due to Győry, and efficient algorithms for several classes of number fields, mainly those given by Gaál, Győry, Pethő, Pohst and their collaborators. Recently, many authors based their approach on prime ideal factorization via Newton polygon techniques as introduced by Ore, and developed by Guàrdia, Montes and Nart. They studied the problem of the monogenity and indices in various families of number fields. In my talk, I will begin by recalling some fundamental results regarding monogenity, not monogenity, and indices in number fields. Following that, I will present some definitions and results concerning the application of Newton polygon techniques in the decomposition of primes in number fields. Then I will provide an overview of my recent results on this topic, focusing on some infinite parametric families of pure number fields $\mathbb{Q}(\sqrt[n]{m})$, and number fields defined by irreducible trinomials of the type $x^n + ax^m + b$. This talk is partly based on joint works with Boudine, Didi, El Fadil and Teibekabe.