Online Number Theory Seminar

7 February 2025. − 17:00-17:50

K. Gyarmati: Combinatorics and Diophantine equations

In this talk, we apply combinatorics to study problems related to Diophantine-type equations. In the first part, we study the cardinality of such sets of squares in which the difference between any two squares is also a square. Such a set with m elements is called a Diophantine square m-tuple. For example, we will see that there are infinitely many Diophantine square triples. It is also proved that there is no Diophantine square triple that only contains squares of Fibonacci numbers. The second part of the talk is a joint work with my colleague Katalin Fried. We say that a set $\mathcal{B} \subseteq \mathbb{Z}$ forms a multiplicative basis of order h of $\mathcal{S}$ if every element of $\mathcal{S}$ can be written as the product of h members of $\mathcal{B}$. We give non-trivial lower bounds for the size of multiplicative basis of order 2 of the set $\{f(1), f(2), \ldots, f(n)\}$ where $f(x) \in \mathbb{Z}[x]$ is a polynomial. We also study generalizations of these problems.