

Online Number Theory Seminar

6 December 2024. – 17:00-17:50

J. Cremona: On the equivalence of binary forms over a field

We consider the question of determining whether two binary forms of small degree over a field K are equivalent under the actions of either $GL(2,K)$ or $SL(2,K)$. In the case of cubics, we give two necessary and sufficient criteria for such equivalence, one of which involves an algebraic invariant, the "Cardano invariant", which belongs to a quadratic extension of K ; this is closely connected to classical formulas, and also similar to an invariant that appears in the work of Bhargava et al. The second criterion for cubics is in terms of the base field itself, and also gives explicit matrices in $SL(2,K)$ or $GL(2,K)$ transforming one cubic into the other, if any exist. These results may be also used to test equivalence of binary cubic forms over an integral domain such as \mathbb{Z} . We will mention a connection between binary cubic forms and the arithmetic of elliptic curves, and discuss similar results for quartic forms, which have application to 2-descent on elliptic curves. The methods are elementary.