Online Number Theory Seminar

19 January 2024. – 17:00-17:50

Yu. Bilu: Skolem meets Schanuel

A linear recurrence of order r over a number field K is a map $U : \mathbb{Z} \to K$ satisfying a relation of the form

$$U(n+r) = a_{r-1}U(n+r-1) + \ldots + a_0U(n) \quad (n \in \mathbb{Z}),$$

where $a_0, \ldots, a_{r-1} \in K$ and $a_0 \neq 0$. A linear recurrence is called simple if the characteristic polynomial $X^r - a_{r-1}X^{r-1} - \ldots - a_0$ has only simple roots, and non-degenerate if λ/λ' is not a root of unity for any two distinct roots λ, λ' of the characteristic polynomial. The classical Theorem of Skolem-Mahler-Lech asserts that a non-degenerate linear recurrence may have at most finitely many zeros. However, all known proofs of this theorem are non-effective and do not produce any tool to determine the zeros.

In this talk I will describe a simple algorithm that, when terminates, produces the rigorously certified list of zeros of a given simple linear recurrence. This algorithm always terminates subject to two celebrated conjectures: the p-adic Schanuel Conjecture, and the Exponential Local-Global Principle. We do not give any running time bound (even conditional to some conjectures), but the algorithm performs well in practice, and was implemented in the Skolem tool
`https://skolem.mpi-sws.org/`
that I will demonstrate.

A joint work with Florian Luca, Joris Nieuwveld, Joël Ouaknine, David Purser and James Worrell.