

Debrecen Number Theory Seminar

5 May 2023. – 13:00-13:50

Location: M402

Gergő Batta and Márton Szikszai: Higher power rational Diophantine tuples

A k th power rational Diophantine n -tuple is a set of n nonzero rationals such that the product of any two distinct raised by one is a rational k th power. A natural question to ask is the following: how large such a set can be? The classical branch of the problem deals with rational integers and squares and has seen a considerable amount of progress that culminates in, if we may claim, the proof of the so-called Diophantine quintuple folklore conjecture stating that there exists no such quintuple. Relaxing the requirements by allowing proper rational numbers in the tuples, we find that there is a rich literature of related results. Interestingly enough, the case of general exponents, or even just a particular one that is different than 2, attracted scarce attention, and only has sporadic mentions and limited understanding.

In this talk, we attempt to lay a foundation for, or just simply promote, future efforts in the study of higher power rational Diophantine tuples. Szikszai will recall one of his earlier talks at the seminar and presents it in a more historical coating, focusing on an experimental and naive approach leading to various obstacles to be overcome. Batta will present an arithmetic geometric framework to attempt a breakthrough for the case of cubic powers. This leads to both a constructor method for cubic rational Diophantine triples by means of extending pairs and an application of this in finding infinite families of cubic triples and quadruples. We also plan to lay out some of the short term challenges to be taken on in the research project, provided that time allows.