Alina Ostafe: Integer matrices with a given characteristic polynomial and multiplicative dependence of matrices

We consider the set $\mathcal{M}_n(\mathbb{Z}; H))$ of $n \times n$-matrices with integer elements of size at most H and obtain upper and lower bounds on the number of s-tuples of matrices from $\mathcal{M}_n(\mathbb{Z}; H)$, satisfying various multiplicative relations, including multiplicative dependence, commutativity and bounded generation of a subgroup of $\mathrm{GL}_n(\mathbb{Q})$. These problems generalise those studied in the scalar case $n = 1$ by F. Pappalardi, M. Sha, I. E. Shparlinski and C. L. Stewart (2018) with an obvious distinction due to the non-commutativity of matrices. As a part of our method, we obtain a new upper bound on the number of matrices from $\mathcal{M}_n(\mathbb{Z}; H)$ with a given characteristic polynomial $f \in \mathbb{Z}[X]$, which is uniform with respect to f. This complements the asymptotic formula of A. Eskin, S. Mozes and N. Shah (1996) in which f has to be fixed and irreducible.

Joint work with Igor Shparlinski.