Online Number Theory Seminar

28 April 2023. – 17:00-17:50

Ankita Jindal: Irreducibility and Galois groups of Laguerre Polynomials

Let α and n be integers with $n \geq 1$. We shall discuss the problem of irreducibility of Generalized Laguerre Polynomial $L_n^{(\alpha)}(x)$ with degree n and parameter α defined by

$$L_n^{(\alpha)}(x) = \sum_{j=0}^{n} (-1)^{n-j} \frac{(n+\alpha)(n-1+\alpha)\cdots(j+1+\alpha)}{(n-j)!j!} x^j.$$

When $\alpha = -n-1$, $L_n^{(-n-1)}(x) = \sum_{j=0}^{n} \frac{x^j}{j!}$ is the nth Taylor polynomial of the exponential function. In 1930, I. Schur proved that $L_n^{(-n-1)}(x)$ is irreducible over the field $\mathbb{Q}$ of rational numbers and its Galois group over $\mathbb{Q}$ is the alternating group A_n or symmetric group S_n of degree n according as $n \equiv 0 \pmod 4$ or not. We describe classes of the polynomials $L_n^{(\alpha)}(x)$ when these polynomials are irreducible over $\mathbb{Q}$. The irreducibility is established using Newton polygons which will also be defined in the talk and their main properties together with some applications will be mentioned. This talk is partly based on joint work with Shanta Laishram, Saranya Nair, Ritumoni Sarma and Tarlok Nath Shorey.