Online Number Theory Seminar

11 November 2022. – 17:00-17:50

Attila Pethő: Common values of a class of linear recurrences

Let $(a_n), (b_n)$ be linear recursive sequences of integers with characteristic polynomials $A(X), B(X) \in \mathbb{Z}[X]$ respectively. Assume that $A(X)$ has a dominating and simple real root α, while $B(X)$ has a pair of conjugate complex dominating and simple roots $\beta, \bar{\beta}$. Assume further that α/β and $\bar{\beta}/\beta$ are not roots of unity and $\delta = \log|\alpha|/\log|\beta| \in \mathbb{Q}$. Then there are effectively computable constants $c_0, c_1 > 0$ such that the inequality

$$|a_n - b_m| > |a_n|^{1-(c_0 \log^2 n)/n}$$

holds for all $n, m \in \mathbb{Z}^2_{\geq 0}$ with $\max\{n, m\} > c_1$. We present c_0 explicitly.
We present two infinite families of linear recursive sequences, which satisfy the assumptions of the theorem.
As a byproduct we prove explicit bounds for the parameters appearing in the Binet formula for linear recursive sequences.